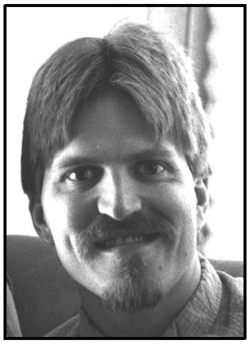
WIND INSTABILITY

WHAT BARROWMAN LEFT OUT



by
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In Centuri TIR-33 (reprinted in the March '98 issue of *High Power Rocketry*), Jim Barrowman outlined a method for the determination of the center of pressure (CP) of a model rocket, now known as the Barrowman Equations (BEq). He recognized that the CP moves forward as the angle of attack (AOA) increases from zero. The largest AOA experienced by a model rocket is when it leaves the launch rod in windy conditions. The larger the wind, the larger the apparent AOA of the rocket. It is usually assumed that a 1 caliber (rocket diameter) margin between the CP and the center of gravity (CG) provides sufficient margin for this forward motion of the CP to allow for a stable flight of a model rocket.

In the article "Wind-Caused Instability" in the same HPR issue, Bob Dahlquist presents experimental results on this CP variation for four model rockets. This data shows a linear variation of CP vs. AOA. This deviation is significant even at angles less than, 10°, the region that Barrowman considered small. The Alpha, a shorter/squat, rocket shows a smaller CP variation (in terms of calibers) than the longer/skinnier rockets (Nike-smoke and Delta Clipper).

An extension to the Barrowman Equations that models this CP variation with AOA is presented here. This extension well models the CP variation for three of the rockets measured by Dahlquist (The fourth rocket had canted fins and did not fit the assumptions of the BEq). Also presented are some predictions for two extreme cases. One is a long-skinny rocket that went unstable at a CMASS launch this spring. This example shows dramatically that the one caliber rule of thumb is not sufficient for stable flight in all cases. The other is the Estes Fatboy which indicates that short/fat rockets may be more stable than typically thought.

What Barrowman Left Out

In Centuri TIR-33, a plot of body lift vs. AOA shows that this force is quite small at angles less than 10°. This plot is used to justify the neglect of body lift in the BEq. However, at these small angles, the wing and nose lift varies linearly with AOA, which also falls to zero at small angles so that it is not clear at what point the body lift can be neglected. The body lift force [references 1,2,3,4] may be expressed, for small angles, as:

$$N = \frac{1}{2}\rho V^2 K A_p \alpha^2$$

where K is a constant between 1.1 and 1.5, $\frac{1}{2}\rho V^2$ is the dynamic pressure, A_p is the body planform area (including the nose, body and all transitions and boattails but not the fins) and α is the angle of attack, measured in radians. This lift acts at the center of the planform area. When this force is put into the BEq format, one factor of α is factored out, leaving a linear variation with angle. This is just what is required to give the linear variation of CP vs. AOA found experimentally. Put into the Barrowman format, the coefficient of body lift is:

$$C_{N\alpha^2} = 4\frac{K}{\pi}\frac{A_p}{D^2}\alpha$$

Where D is the diameter of the rocket at the base of the nose cone. This force acts at the center of planform area:

$$\overline{X_B} = X_{\text{plan}}$$

The contribution of each body component (nose, body tube, transitions) can be calculated separately or the entire body lift contribution can be done at once using the total planform area. Table 1 shows the planform area and moment arm for typical model rocket components shown in Figure 1.

	A_{plan}	X_{plan}
Conical Nose	$\frac{1}{2} L_N D$	$\frac{2}{3} L_N$
Parabolic Nose	$\frac{2}{3} L_N D$	$\frac{3}{5} L_N$
Ogive Nose	$\frac{2}{3} L_N D$	$\frac{5}{8} L_N$
Cylindrical Body	$L_B D_B$	$X_{BN} + \frac{1}{2} L_B$
Conical Transition	$\frac{1}{2} (D_1+D_2) L_T$	$X_{TN} + \frac{1}{3} L_T (D_1+2D_2)/(D_1+D_2)$

Table 1: Area and Moment Arm for Rocket Components

The force from transitions is positive, regardless of whether it is an expanding or reducing transition, since it only depends on area.

To apply these equations, simply add $C_{N\alpha^2}\overline{X_B}$ to the numerator and $C_{N\alpha^2}$ to the denominator of the usual BEq for each rocket component.

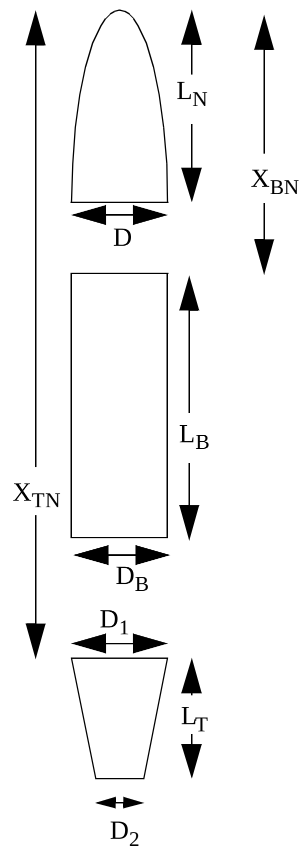

Figure 1: Rocket Components

Comparison of Model with Data

I received the dimensions for three of the four rockets measured in the HPR stability article from Konrad Hambrick, an Alpha II, Nike-Smoke and Delta Clipper (Figure 2).

I applied the BEq and my body lift extension to these rockets. The constant K was varied to fit the three data sets. A value of 1.0 gave good agreement between the data and the model. Table 2 presents the resultant CP equations in terms of calibers. The model predictions and experimental data are presented in Figure 3. The data is show by dashed lines and the model by solid lines.The agreement between the model and experiment is near perfect up to about 10° and quite good up to 15°. Above 15°, the actual CP moves forward more quickly than predicted by this model. This behavior is prob-

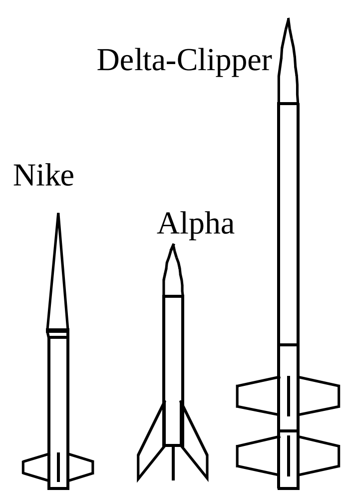

Figure 2: Test Rocket Planforms

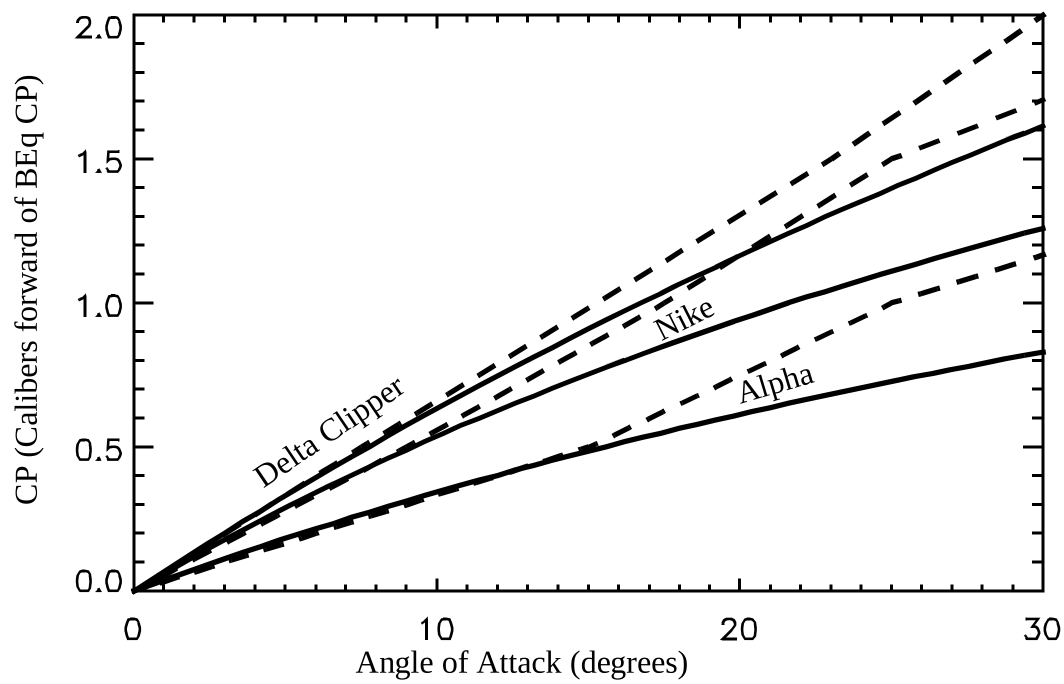

Figure 3: Experimental vs. Predicted CP Variation

Model	CP Equation
Alpha II	$CP = \dfrac{127.4 + 67.4\alpha}{14.7 + 11.6\alpha}$
Nike-Smoke	$CP = \dfrac{168.2 + 110.2\alpha}{13.7 + 13.0\alpha}$
Delta Clipper	$CP = \dfrac{1029.6 + 376.1\alpha}{50.7 + 28.5\alpha}$

Table 2: CP vs. AOA for the Test Rockets

ably due to some other BEq assumption being violated, such as the fins stalling. These results indicate that this method of adding body lift into the BEq is a good approximation.

In extracting the experimental points from Bob Dahlquists's article, I left out one point which deviated substantially from the trend of the rest of the data for the Delta Clipper just as Bob did when he drew lines through his data. Using a larger value for K of about 1.2 would give a better match at large angles but slightly worse at small angles.

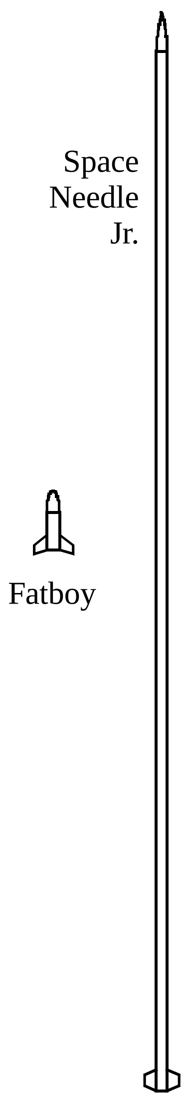

Figure 4: Two Extreme Rockets

Other Predictions

The three rockets discussed so far have been relatively "normal" rockets. It is interesting to examine two extreme casesin terms of aspect ratio, a very long/skinny rocket and a short/fat one to see how the CP moves in these cases. These two rockets are shown in Figure 4.

At a recent CMASS launch, Sean Lannan launched a Rogue Aerospace Space Needle Jr. that he had flown successfully before on ½A and A engines. On this particular day, he used a B and the model went unstable. Applying the BEq to this rocket gave stability margins of 20, 16 and 12 calibers for the ½A, A and B engines respectively. That would appear to be quite sufficient for all of the engines. However, since the body is quite large relative to the fins, the body lift is very important even at relatively small angles of attack. The variation of this rocket's CP with AOA is shown in Table 3 and is plotted in Figure 5A (Note the 15 times scale change from Figure 3). This rocket loses over 12 calibers of stability at only 5° AOA! Clearly here, the body lift is very important and the usual 1 caliber rule of thumb is quite insufficient for stable flight even in relatively light winds. This also shows why the ½A and A flights could be stable but the B went unstable.

Model	CP Equation
Space Needle Jr.	$CP = \dfrac{886.8 + 4107.6\alpha}{13.2 + 100.3\alpha}$
Fatboy	$CP = \dfrac{188 + 19.2\alpha}{40.5 + 6.2\alpha}$

Table 3: CP vs. AOA for Two Extreme Rockets

The CP movement of a short/fat rocket, the Estes Fatboy is and is plotted in Figure 5B. Note that in this plot, the full scale is only 1 caliber. Even at 30° the CP moves forward less than ⅕ caliber. Clearly here, the full 1 caliber stability margin is not necessary even in relatively high winds.

This CP prediction may be used to estimate the maximum allowable winds to allow stable flight for a given rocket/motor combination. First use this method to predicts CP vs. angle of attack. Measurement of the CG location then gives a maximum angle of attack, α_{max}, where the CP equals the CG. If the rocket velocity as it leaves the launch rod, V_{launch}, can be estimated, then a maximum wind velocity for stable flight can be found: $V_{max} = V_{launch} \tan\alpha_{max}$. This can be done through a wRASP-like altitude prediction code or by estimating the initial acceleration of the rocket through the initial motor thrust and rocket mass. There obviously still needs to be some CP-CG margin but what minimum value is acceptable is not clear. A more involved simulation including the actual turning moment and moment of inertia would be required to answer this question.

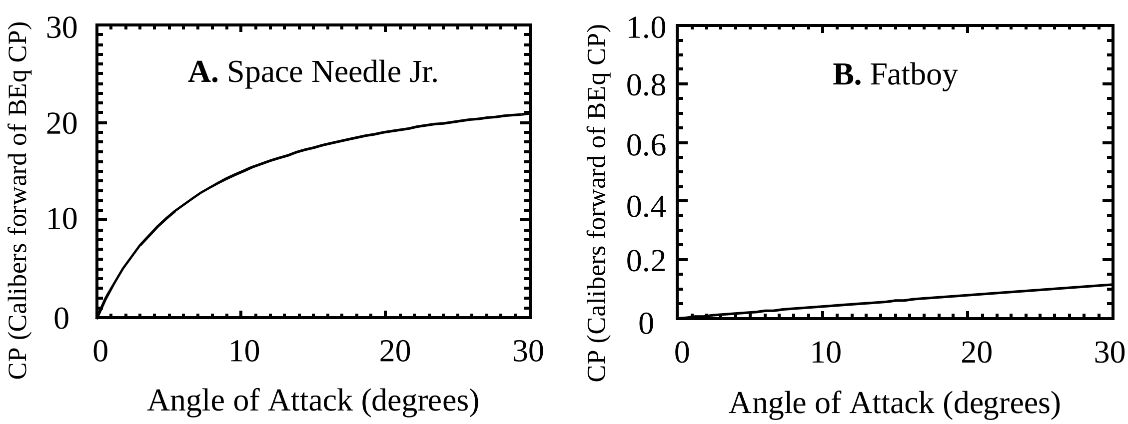

Figure 5: CP Variation for Two Extreme Rockets

Summary

An extension to the Barrowman equations was presented that includes the effects of body lift. This extension fits Bob Dahlquist's experimental results quite well and explains an unstable flight of a long/skinny rocket. Using this extension increases the ability to predict CP out to about 15° angle of attack. For "normal" rockets, the one caliber stability rule of thumb appears to be a good rule of thumb. However, for long/skinny rockets, upwards of ten calibers may be called for and for short/fat rockets less than half a caliber may be sufficient.

A VCP-like CP prediction code with this extension combined with a wRASP-like rocket flight prediction code could be used to predict a maximum acceptable wind speed for stable launch of a given rocket/motor combination.

References

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